Holographic Modelling of SCGT

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Work inspired by Johanna Erdmenger, Kostas Rigatos and Werner Porod: 1907.09489 [hep-th]
2009.10737 [hep-ph]
2304.09190 [hep-th]
2404.14480 [hep-ph]

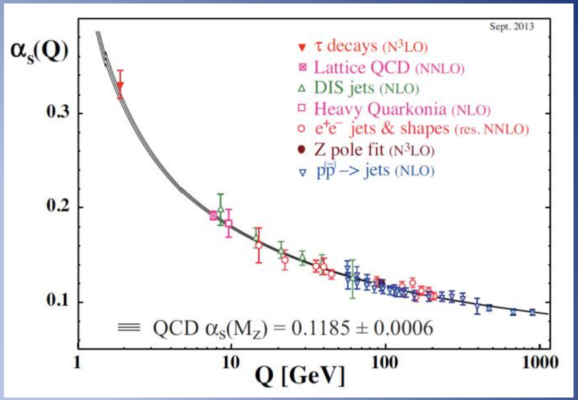
New results with Anja Alfano
2409.07977 [hep-ph]
2412.07309 [hep-ph]

Southampton Theory Astrophysics Gravity Research Centre

- Holographic modelling
- Walking dynamics
- Composite higgs models
- Splitting confinement and chiral symmetry breaking

Strongly Coupled Gauge Theories

QCD (SU(3) with Nf fundamental rep. quarks) at strong coupling

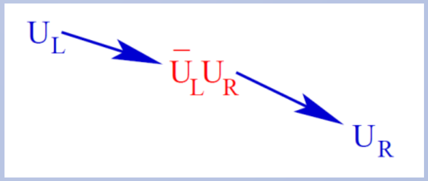


- Confines

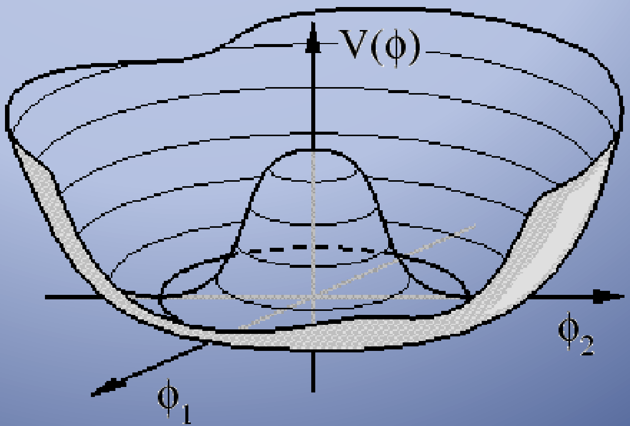
  Localized, magnetically charged, scalar gauge configurations form and condense leading to a dual Meissner effect (?)….

- Breaks chiral symmetry – mass generation mechanism

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$



$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. \rangle \neq 0$$

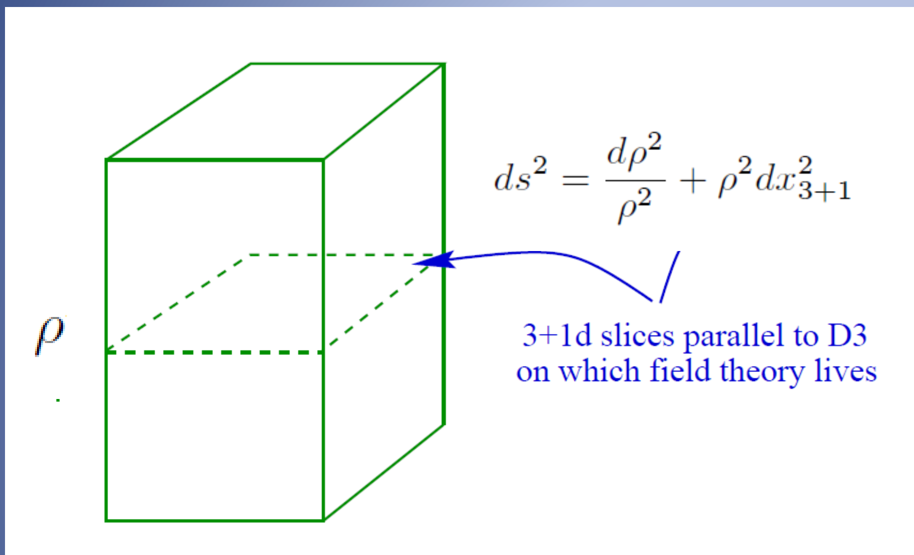


Are these behaviours generic and all SCGT the same?

AdS/CFT

Holography emerged from string theory

It is a remarkable strong weak duality that describes SCGT with weakly coupled gravity



Dilatations

$$\int d^4x \; \partial^\mu \phi \partial_\mu \phi, \qquad x \to e^{-\alpha} x, \qquad \phi \to e^{\alpha} \phi$$

Become spacetime symmetry of AdS

$$\rho \to e^{\alpha} \rho$$

ρ is a continuous mass dimension $\to$ RG Scale

We have control in theories with large amounts of SUSY

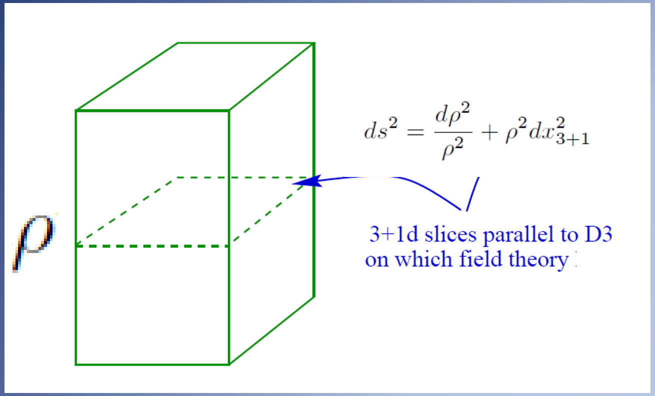
But we can describe those theories at finite temperature

The breaking of SUSY and conformality are intrinsically built into the framework.

RG flow is represented by a (holographic) radial direction in the space

Classical equations describe the RG flow of operators and sources.

How Does AdS/CFT Work 2



Operators and sources appear as fields in the bulk

Eg

$$\int d^4x \; m \; \bar{\psi}\psi$$

m is the quark mass
c is the quark condensate

The duality maps the mass of a scalar in AdS to the dimension of the operator it is dual to

$M^2 = \Delta (\Delta-4)$

Scalars in AdS become unstable when

$M^2 < -4/R$

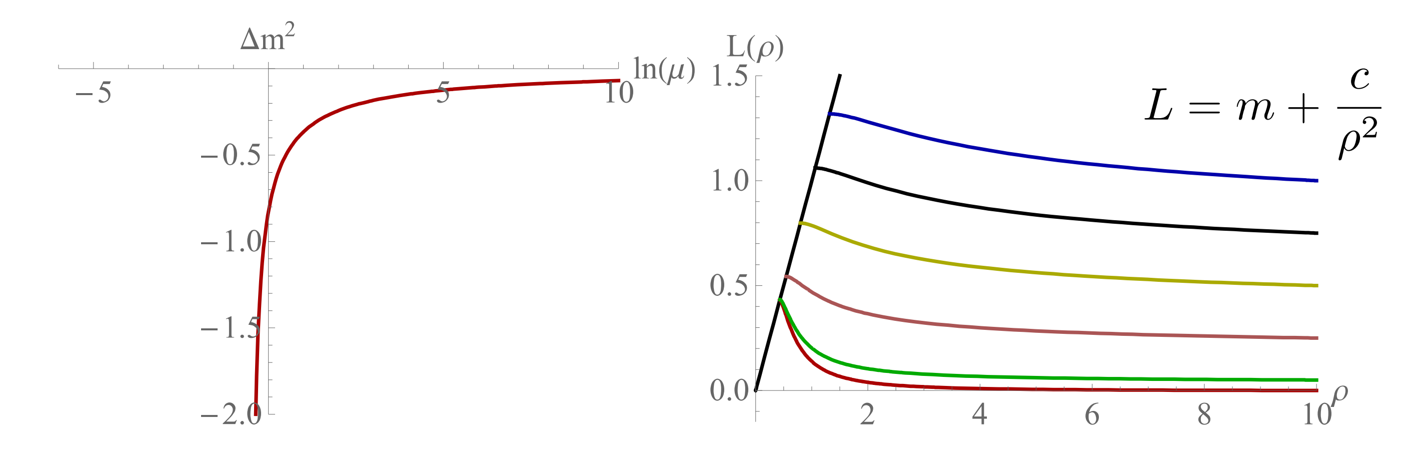
Simple models of chiral symmetry breaking have

$\Delta = 3$ in UV running to $\Delta = 2$ in the IR when a higgs mechanism occurs in the bulk

Formation of the Chiral Condensate

We solve for the vacuum configuration of L

$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0\,.$$



Read off m and qq in the UV…

Δm^2 from QCD

Shoot out with

$L'(\rho = L) = 0$

The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

No AF

Nf=11/2 Nc

CFT

Walking

Nf = 4 Nc

χSB

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

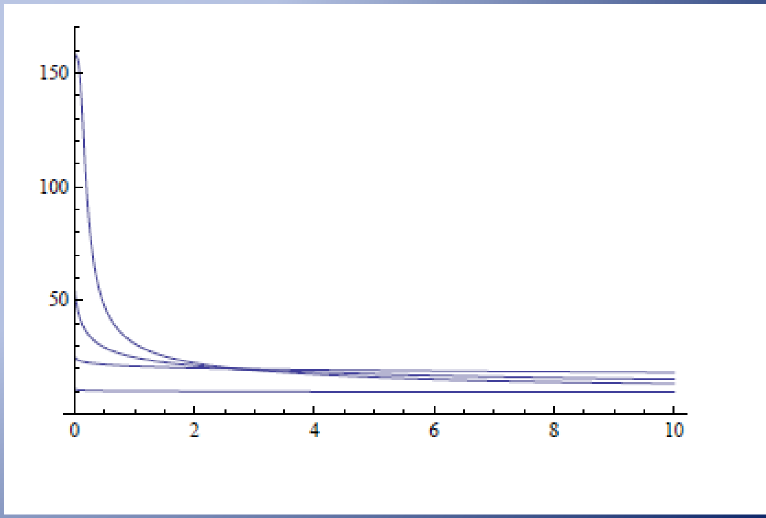
$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$

m $\bar{q}q$

$$\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3(N_c^2-1)}{4N_c\pi}\alpha$$

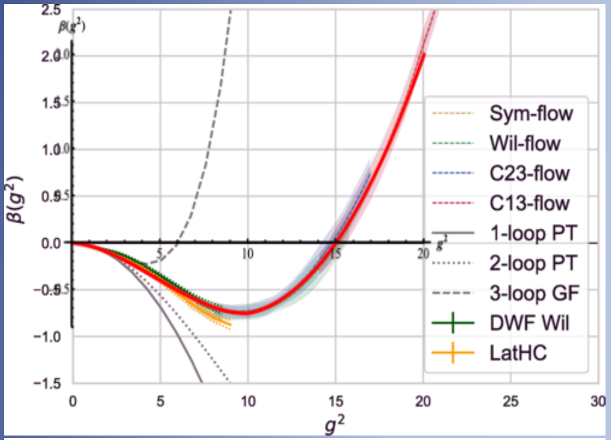


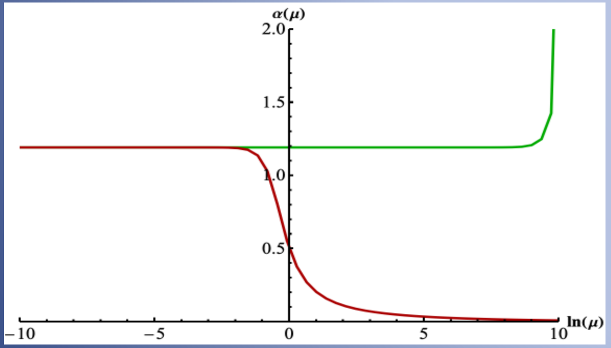
If critical γ = 1…. Nf/Nc ~ 4

Yamawaki,Appelquist, Terning, Sannino,…

Walking Dynamics

A lattice study (28^3x56) at Nf=10 concludes the theory is in the conformal window





[1] A. Hasenfratz, E. T. Neil, Y. Shamir, B. Svetitsky and O. Witzel, Phys. Rev. D **108** (2023) no.7, L071503 doi:10.1103/PhysRevD.108.L071503 [arXiv:2306.07236 [hep-lat]].

Fixed point at g^2=15

qq dimension = 2.4

The holographic model lets us play in the space of theories to understand this result better…

2412.07309. (Alfano, NE)

What if the instability to chiral symmetry breaking lies below g^2=15?

$$\Delta m^2 = -k\alpha,$$

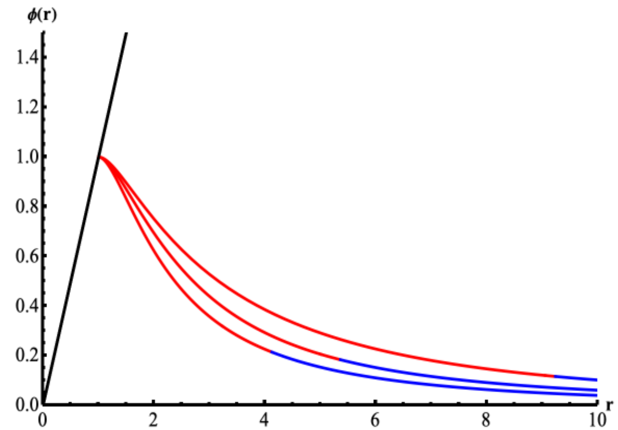
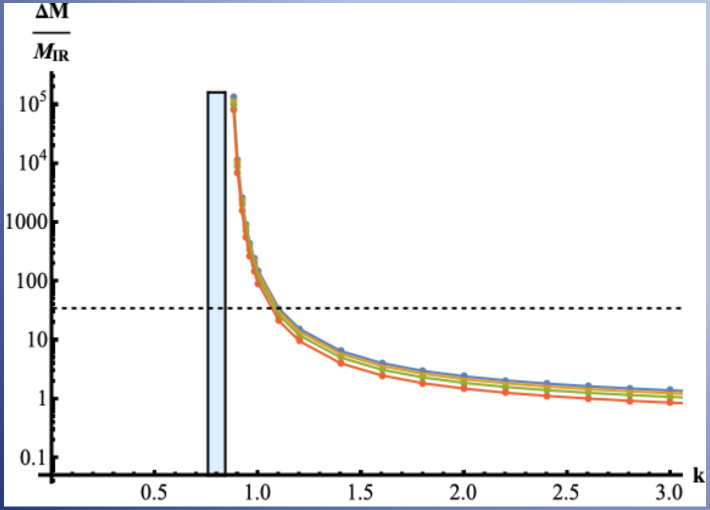


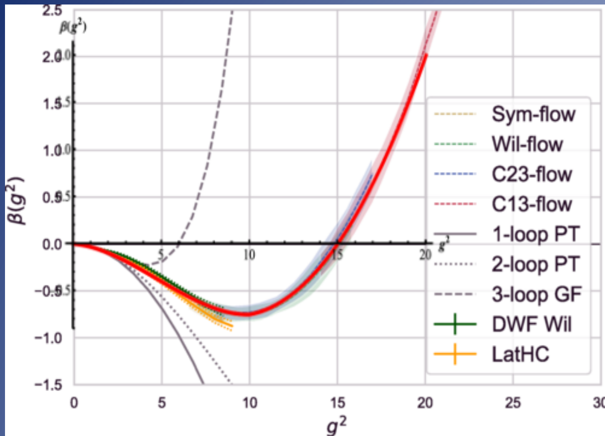
Fig 2: The vacuum solutions ϕ for $k = 3, 1.8, 1.4$ with the scale of $r_{\rm min}$ scaled to one in each case. The red part of each solution shows the r range where the BF bound is broken at $\phi = 0$.

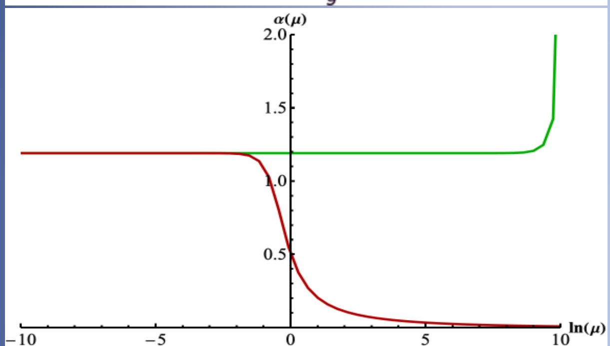
There's the walking gap between entering strong coupling and triggering chiral symmetry breaking…

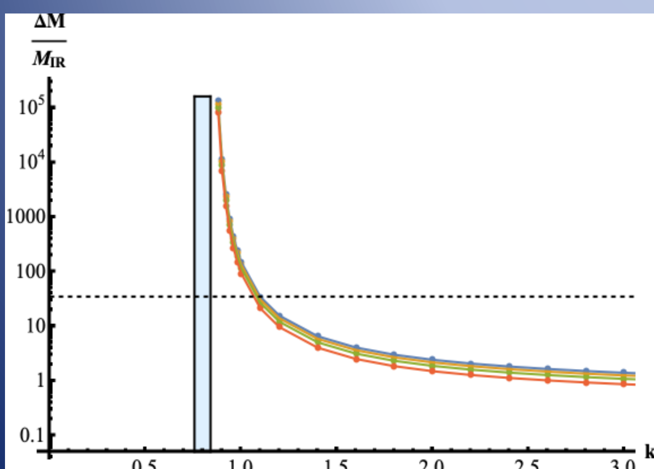
But also if the BF bound violation is weak then a further gap between the condensate scale and the BF bound violation scale sets in…



For a good chunk (10-15%) of this range the condensate lies below the lattice scale range even if you set the UV coupling to be 95% of the fixed point value….







But the lattice guys are smart.. They also ran with a coupling above the fixed point value and saw a conformal IR…

The lattice protects you from the UV Landau pole…

Can you always do this? NO.

At the edge of the CW the IR coupling violates the BF bound so any value above the fixed point will break chiral symmetry at the UV scale – the “lattice artefact phase”

One could have a theory on the lattice that is conformal from below and chiral symmetry breaking from above unless you massively fine tune – the blue area is 10%...

Splitting Scales

Alfano + NE. 2409.07977 [hep-ph]

SCGT with fermions in two representations may have split mass scales.

Larger reps than the fundamental typically are more strongly coupled by group theory factors – will they condense and become massive at higher scales than fundamentals?

The problem is the running is usually fast at strong coupling.

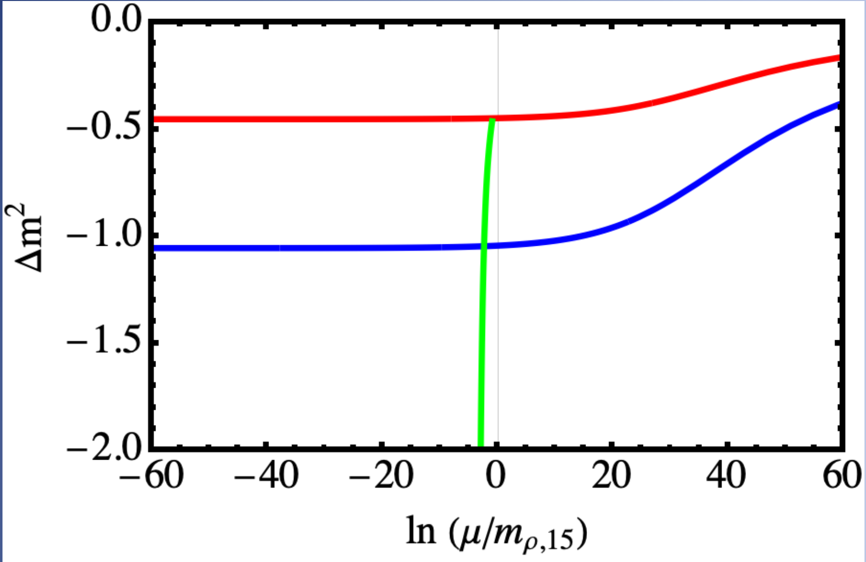
We try to insert walking between the scales…

EG SU(Nc) + Nf fundamentals
+ 1 Dirac two index symmetric rep

$SU(5)$ gauge theory with $N_f^F = 15$

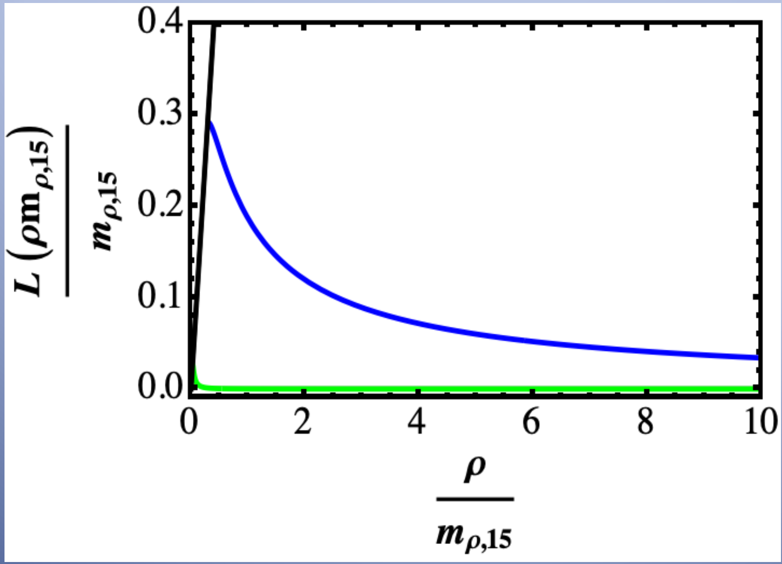
+ one 15 dim rep

$$\Delta m^2 = -2\gamma.$$



$$\mu \frac{d\alpha}{d\mu} = -b_0\alpha^2 - b_1\alpha^3 \,,$$

$$\begin{aligned} b_0 &= \tfrac{1}{6\pi}\left(11C_2(G) - 4\textstyle\sum_R T(R)N_f(R)\right)\,, \\ b_1 &= \tfrac{1}{24\pi^2}\Big(34C_2^2(G) \\ &\quad - \textstyle\sum_R \left(20C_2(G) + 12C_2(R)\right)T(R)N_f(R)\Big) \end{aligned}$$



It is also possible to compute the meson spectrum of the theory by looking at plane wave fluctuations about the background

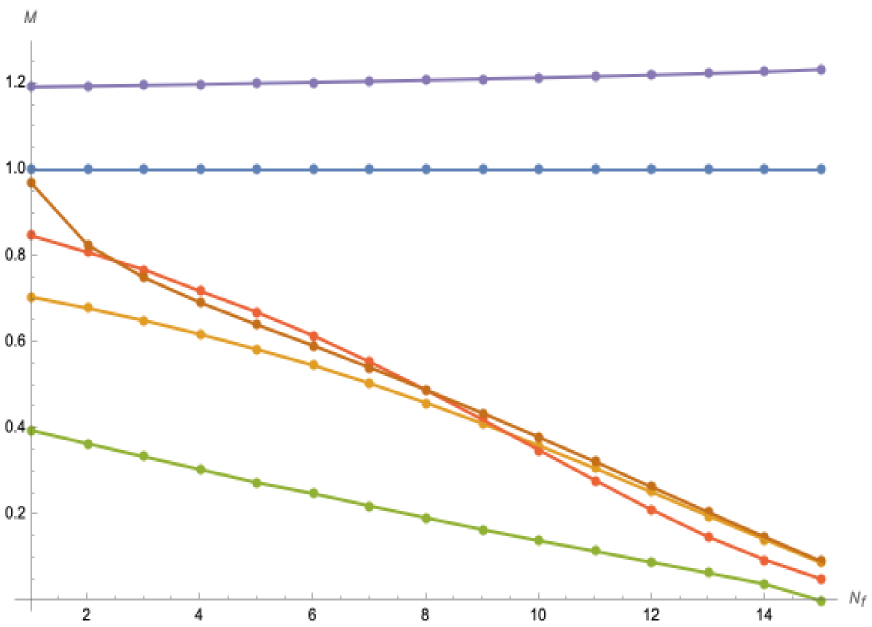


Fig 3: Mass spectra for the SU(5) theory, ρ mesons in blue (symmetric) and dark yellow (fundamental), σ mesons in green (symmetric) and orange (fundamental), axials in purple (symmetric) and brown (fundamental). The pions in both sectors are massless at zero fermion mass.

We see a gap grow with increasing Nf between the 15 sector and the 5…

The (green) sigma meson of the 15 sector is very light reflecting the running dynamics leading to a light dilaton like state….

Gaps can be bigger than 10 (in this model) but those theories are very walking… these might be more accessible…

$$\mathrm{SU}(3),\ N_f^F = 8:\ \frac{m_{\rho 15}}{m_{\rho F}} = 7.34$$
$$\mathrm{SU}(4),\ N_f^F = 8:\ \frac{m_{\rho 15}}{m_{\rho F}} = 2.93$$
$$\mathrm{SU}(5),\ N_f^F = 12:\ \frac{m_{\rho 15}}{m_{\rho F}} = 3.97$$
$$\mathrm{SU}(6),\ N_f^F = 16:\ \frac{m_{\rho 15}}{m_{\rho F}} = 5.18$$
$$\mathrm{SU}(7),\ N_f^F = 20:\ \frac{m_{\rho 15}}{m_{\rho F}} = 6.59$$

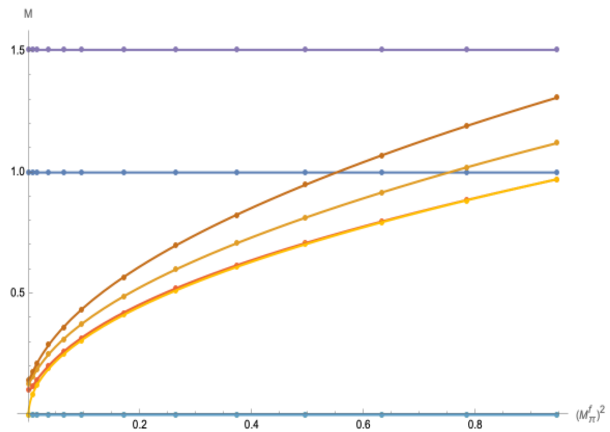


Fig 8: Mass spectra for the SU(3) theory with $N_f = 8$ and non-zero UV mass. ρ mesons in blue (symmetric) and dark yellow (fundamental), σ mesons in green (symmetric) and orange (fundamental), axials in purple (symmetric) and brown (fundamental) and pions in cyan (symmetric) and light yellow (fundamental).

The gap closes if the fundamental mass is too large…

The gaps depend on the choice of critical coupling… and we've neglected interactions between the sectors… it will need lattice computations to prove a gap…

Is this a gap to confinement also?

SU(4) 3 F 3 $\bar{F}$ 5 A_2

G. Ferretti, "UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group," JHEP **06** (2014) 142, arXiv:1404.7137 [hep-ph].

In this model the A2 symmetry breaking generates the SM higgs and the Fs are to give F A_2 F top partners

The lattice has simulated (unquenched) SU(4) 2 F 2 F 4 A_2

	Lattice [80] $4A_2, 2F, 2\bar{F}$ unquench	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ no decouple	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ no decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
M_{VA_2}	1.00(4)	1*	1*	1*	1*	1*
f_{VA_2}	0.68(5)	0.489	0.489	0.516	0.516	0.517
M_{VF}	0.93(7)	0.933	0.939	0.890	0.904	0.976
f_{VF}	0.49(7)	0.458	0.461	0.437	0.491	0.479
M_{AA_2}		1.37	1.37	1.32	1.32	1.28
f_{AA_2}		0.505	0.505	0.521	0.521	0.522
M_{AF}		1.37	1.37	1.21	1.23	1.28
f_{AF}		0.501	0.504	0.453	0.509	0.492
M_{SA_2}		0.873	0.873	0.684	0.684	1.18
M_{SF}		1.03	1.02	0.811	0.798	1.25
M_{JA_2}	3.9(3)	2.21	2.21	2.21	2.21	2.22
M_{JF}	2.0(2)	2.07	2.08	1.97	2.00	2.17
M_{BA_2}	1.4(1)	1.85	1.85	1.85	1.85	1.86
M_{BF}	1.4(1)	1.74	1.75	1.65	1.68	1.81

The pattern is right…

The A2-F gap is very well described…

Adding extra flavours is not a huge change…

Scalar masses get lighter as add extra flavours

Summary & Future

- Holographic models neatly encapsulate RG flow and the instability at strong coupling to condensation

- This is a quick tool to get intuition for walking theories – very hard to simulate on the lattice when within 10% of the CW edge.

- Supports hints in lattice data of scale gaps between different representations

- We are looking at other theories eg with Adjoints to map out the gap

- Looking at the interplay with the confinement mechanism via QCD(Adj) on a S1

- A parallel program is mapping out the spectra of possible composite higgs models that also have multi-reps (with Werner and Johanna)

- We hope to map out SCGT dark matter in more detail

Composite Higgs Model Example

NE, Porod, Erdmenger, Liu

Let's focus on an Sp(2Nc) gauge theory with 2 Dirac fundamental fermions.

The pseudo-reality means the flavour symmetry is U(4) on the 4 Weyl fermions (we neglect the anomaly)

$$_i = \begin{pmatrix} U_L^C \\ D_L^C \\ D_R \\ U_R \end{pmatrix}$$

The vacuum condensate is anti-symmetric in spin ($2\mathrm{x}2=1_A+3_S$), anti-symmetric in colour, so anti-symmetric in flavour… possible vacua

$$X = \begin{pmatrix} 0 & L_0 & 0 & 0 \\ -L_0(\rho) & 0 & 0 & \\ 0 & 0 & 0 & -L_0 \\ 0 & 0 & L_0 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 0 & 0 & -Q \\ 0 & 0 & Q & 0 \\ 0 & -Q & 0 & 0 \\ Q & 0 & 0 & 0 \end{pmatrix}$$

$$X \to \begin{pmatrix} 0 & L_0+Q_0 & 0 & 0 \\ -L_0-Q_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_0-Q_0 \\ 0 & 0 & -L_0+Q_0 & 0 \end{pmatrix},$$

$$X' = \Omega X \Omega^T$$

L1, L6

Both break U(4)-> Sp(4)…. The first is $SU(2)_L$ invariant the second breaks $SU(2)_L$

$$X = \begin{pmatrix} 0 & L_0 & 0 & 0 \\ -L_(& 0 & 0 & 0 \\ 0 & 0 & 0 & -L_0 \\ 0 & 0 & L_0 & 0 \end{pmatrix}$$

Biased with $SU(2)_L$ preserving masses

$$M = \begin{pmatrix} 0 & m_1 & 0 & 0 \\ -m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & -m_2 & 0 \end{pmatrix}$$

There are 2x6 real meson fields in here…. Which we parameterize as…

$$X_f = \begin{pmatrix} 0 & \sigma - Q_5 + iS - i\pi_5 & Q_2 - \pi_2 + i\pi_1 - iQ_1 & -Q_4 + \pi_4 + iQ_3 - i\pi_3 \\ -\sigma + Q_5 + i\pi_5 - iS & 0 & Q_4 + \pi_4 + iQ_3 + i\pi_3 & Q_2 + \pi_2 + iQ_1 + i\pi_1 \\ \pi_2 - Q_2 + iQ_1 - i\pi_1 & -Q_4 - \pi_4 - iQ_3 - i\pi_3 & 0 & \sigma + Q_5 + iS + i\pi_5 \\ Q_4 - \pi_4 + i\pi_3 - iQ_3 & -Q_2 - \pi_2 - iQ_1 - i\pi_1 & -\sigma - Q_5 - iS - i\pi_5 & 0 \end{pmatrix}$$

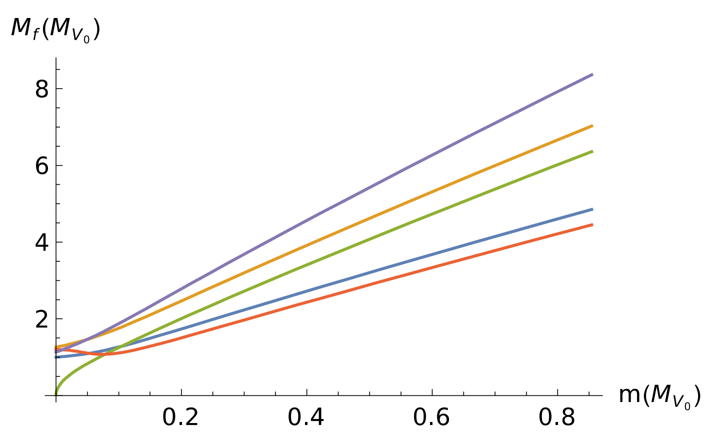
The 6 Goldstones (16 generators -> 10) are π and S

π1-4 are a 4-plet that transform as bi-doublet under SU(2)L x SU(2)R and can be made into a higgs boson… they obtain a mass from m1 and m2… and from loop corrections – W loops and top loops… here concentrate on the strong dynamics…

S (anomaly) and π5 might be dark matter candidates…. Cacciapaglia/Sannino

m=0

Observables	$SU(2)$	Lattice $SU(2)$	$Sp(4)$	Lattice $Sp(4)$	$Sp(6)$	$Sp(8)$	$Sp(10)$
m_V (10)	1^*	1.00(3)	1^*	1.00(33)	1^*	1^*	1^*
m_A (6)	1.66	1.11(46)	1.26	1.61(17)	1.18	1.14	1.12
m_σ (1)	1.26	1.5(1.1)	1.20		1.22	1.23	1.23
m_Q (5)	1.13		1.13		1.13	1.13	1.13
$m_{\pi,S}$ (6)	0.02		0.01		0.01	0.01	0.01
F_V	0.38		0.53	0.52(10)	0.59	0.64	0.67
F_A	0.48		0.54	0.673(92)	0.59	0.63	0.66
f_π	0.06		0.10	0.122 (99)	0.12	0.12	0.13



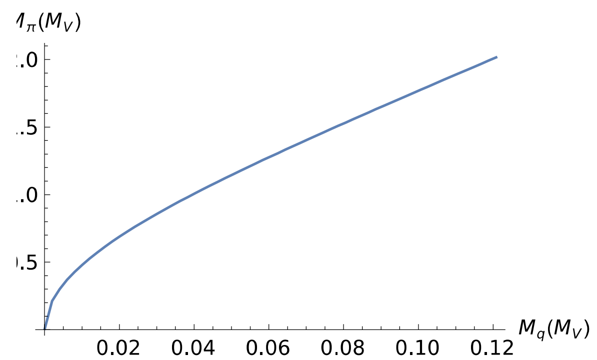


Figure 3: The masses of the bound state f against the UV fermion mass in the unit of M_{V_0}. The gauge group is Sp(4) and $B = 1$.

NJL Competition

$$L = \begin{pmatrix} 0 & L_0 & 0 & -Q \\ -L_0 & 0 & Q & 0 \\ 0 & -Q & 0 & L_0 \\ Q & 0 & -L_0 & 0 \end{pmatrix}.$$

$$\mathcal{L} = \frac{g^2}{\Lambda_{UV}^2}(\bar{\Psi}_L U_R \bar{U}_R \Psi_L + \bar{\Psi}_L D_R \bar{D}_R \Psi_L)$$

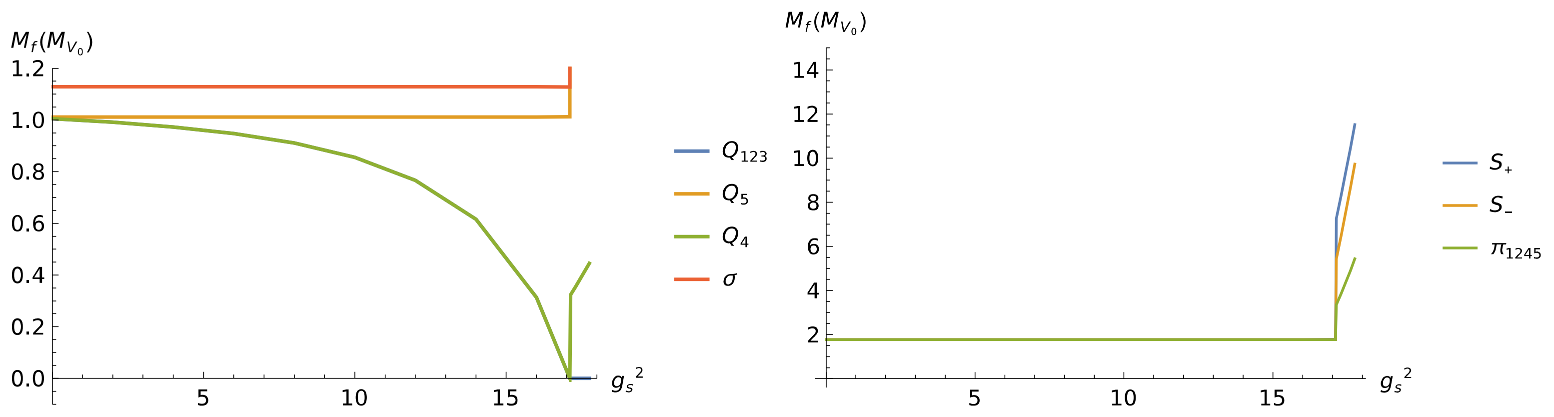
NJL interactions that preserve $SU(2)_L$ favour the Q vacuum…

$$U^T L U = \frac{1}{2}\begin{pmatrix} 0 & -L_0(\rho) - Q(\rho) & 0 & 0 \\ L_0(\rho) + Q(\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & -L_0(\rho) + Q(\rho) \\ 0 & 0 & L_0(\rho) - Q(\rho) & 0 \end{pmatrix}.$$

We now have to change the boundary conditions on the left right states…

$$X_f = \begin{pmatrix} 0 & \sigma - Q_5 + iS - i\pi_5 & Q_2 - \pi_2 + i\pi_1 - iQ_1 & -Q_4 + \pi_4 + iQ_3 - i\pi_3 \\ -\sigma + Q_5 + i\pi_5 - iS & 0 & Q_4 + \pi_4 + iQ_3 + i\pi_3 & Q_2 + \pi_2 + iQ_1 + i\pi_1 \\ \pi_2 - Q_2 + iQ_1 - i\pi_1 & -Q_4 - \pi_4 - iQ_3 - i\pi_3 & 0 & \sigma + Q_5 + iS + i\pi_5 \\ Q_4 - \pi_4 + i\pi_3 - iQ_3 & -Q_2 - \pi_2 - iQ_1 - i\pi_1 & -\sigma - Q_5 - iS - i\pi_5 & 0 \end{pmatrix}$$

The Q1-4 are now exact Goldstones for all Q vevs because SU(2)L is broken explicitly by the vev… we can rotate from composite higgs to technicolour as the NJL g rises…. Paper of spectrum soon! The LHC phenomenology….



(a) M_f^2, $f = Q_{123}, Q_4, Q_5, \sigma$

(b) M_f^2, $f = \pi_{1245}, S_+, S_-$

Figure 9: Scalar bound states' masses vary with increasing scalar NJL coupling g_s^2, with gauge group SU(2), $B = 0.1$, $m_L = 0.12 M_{V_0}$, $\Lambda_{UV} = 12.1 M_{V_0}$.

As the NJL operator switches on a state drives down to an instability and the vacuum rotates to the TC vacuum… the spectrum rapidly grows (apart from Goldstones)…