Entanglement and collider physics

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University of Oxford

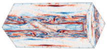
RTG Inauguration Workshop

Würzburg, 17^{th} March 2025

AJB, Phys.Lett.B 825 (2022) 136866 — 2106.01377 [hep-ph]
AJB, P. Caban, J.Rembieliński — 2204.11063 [quant-ph]
R.Ashby-Pickering, AJB, A.Wierzchucka — 2209.13990 [quant-ph]
C.Altomonte, AJB — 2312.02242 [hep-ph]
C.Altomonte, AJB, M.Ekstein, P.Horodecki, K. Sakurai — 2412.01892 [hep-ph]

Review article: AJB, M.Fabbrichesi, R.Floreanini, E.Gabrielli, L.Marzola — 2402.07972 [hep-ph]

Interesting physics $\neq$ 'new' physics $\neq$ beyond-SM physics



ON THE COVER

Heating of Magnetically Dominated Plasma by Alfvén-Wave Turbulence

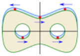
February 14, 2022

Three-dimensional kinetic simulation of the onset of relativistic wave turbulence in the collision of two magnetic shear waves. Selected for a Viewpoint in *Physics*.

Joonas Nättilä and Andrei M. Beloborodov

Phys. Rev. Lett. **128**, 075101 (2022)

Issue 7 Table of Contents | More Covers



Physics NEWS AND COMMENTARY

A Quantized Surprise from Fermi Surface Topology

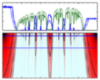
February 16, 2022

The quantized conductance of a two-dimensional electron gas can reflect its Fermi surface topology.

Synopsis on:

C. L. Kane

Phys. Rev. Lett. **128**, 076801 (2022)



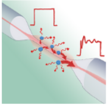
EDITORS' SUGGESTION

Chaotic Diffusion in Delay Systems: Giant Enhancement by Time Lag Modulation

Laminar chaotic diffusion is found in systems with delayed nonlinearity, accompanied by a reduction of the effective dimensionality.

Tony Albers, David Müller-Bender, Lukas Hille, and Günter Radons

Phys. Rev. Lett. **128**, 074101 (2022)



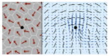
EDITORS' SUGGESTION

Collective Radiative Dynamics of an Ensemble of Cold Atoms Coupled to an Optical Waveguide

An ensemble of cold atoms is coherently coupled in a controlled way to a tapered optical fiber, demonstrating collective effects in this system.

Riccardo Pennetta *et al.*

Phys. Rev. Lett. **128**, 073601 (2022)



Physics NEWS AND COMMENTARY

Extending and Contracting Cells

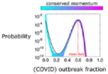
February 15, 2022

Cell-substrate interactions explain a difference in behavior between individual cells and tissues on a surface.

Synopsis on:

Andrew Killeen, Thibault Bertrand, and Chiu Fan Lee

Phys. Rev. Lett. **128**, 078001 (2022)



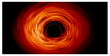
EDITORS' SUGGESTION

Outbreak Size Distribution in Stochastic Epidemic Models

An analytical approach to stochastic epidemic models shows that the statistics of extreme outbreaks depend on an infinite number of minimum-action paths, and that extreme outbreaks define a new class of rare processes for discrete-state stochastic systems.

Jason Hindes, Michael Assaf, and Ira B. Schwartz

Phys. Rev. Lett. **128**, 078301 (2022)



Physics NEWS AND COMMENTARY

Illuminating Black Holes through Turbulent Heating

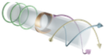
February 14, 2022

Predictions indicate that it should be possible to directly identify how turbulence heats a given black hole's plasma from the spectrum of that plasma's radiation.

Viewpoint on:

Joonas Nättilä and Andrei M. Beloborodov

Phys. Rev. Lett. **128**, 075101 (2022)



Physics NEWS AND COMMENTARY

Waves in a Solid Imitate Twisted Light

February 11, 2022

Waves of vibration moving through the walls of a pipe can carry orbital angular momentum that could be used for several purposes, according to new theoretical work.

Focus story on:

G. J. Chaplain, J. M. De Ponti, and R. V. Craster

Phys. Rev. Lett. **128**, 064301 (2022)

Some of the old problems are amongst the deepest. . .

EINSTEIN ATTACKS QUANTUM THEORY

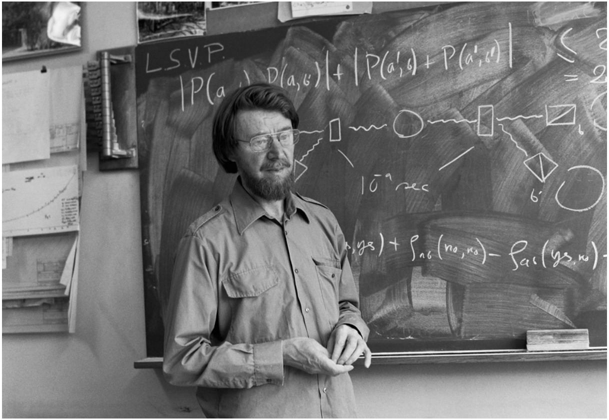
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

New York Times, May 4 **1935**, reporting on Einstein-Podolsky-Rosen paper,
"*Can Quantum-Mechanical Description of Physical Reality Be Considered Complete*"

. . . and they are experimentally accessible



J.S. Bell *'On the Einstein Podolsky Rosen paradox'* (1964)

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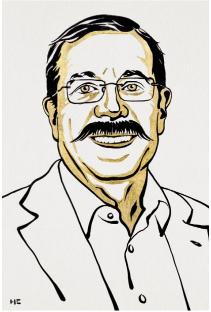
The Nobel Prize in Physics 2022



Ill. Niklas Elmehed © Nobel Prize Outreach

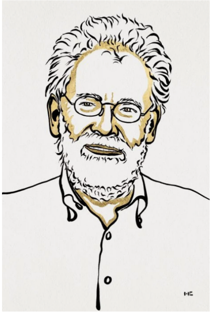
Alain Aspect

Prize share: 1/3

Ill. Niklas Elmehed © Nobel Prize Outreach

John F. Clauser

Prize share: 1/3

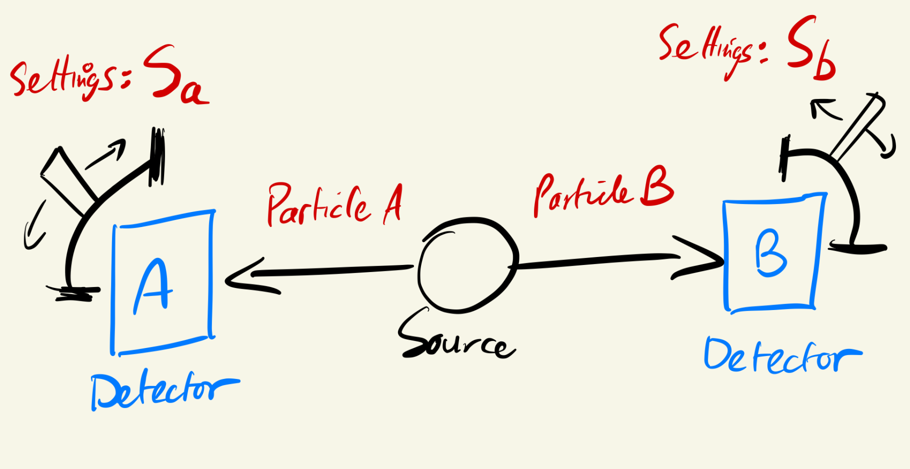


Ill. Niklas Elmehed © Nobel Prize Outreach

Anton Zeilinger

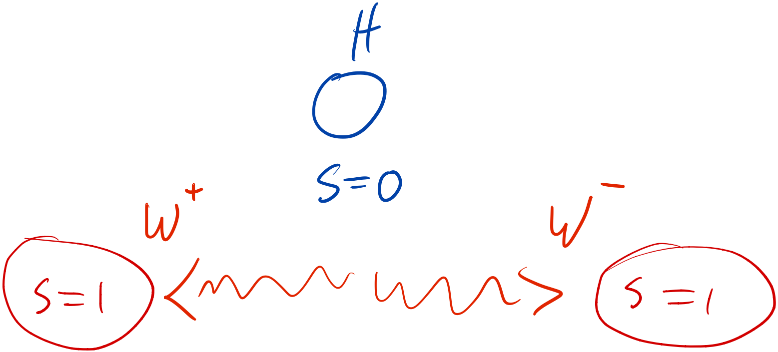
Prize share: 1/3

The textbook case – apparatus



(Ensemble of similarly-prepared systems)

Due to Bohm


H
S=0
W+
W−
S=1
S=1

Spin in the $H \to W^+W^-$ decay

The Higgs boson is a scalar, while $W^{\pm}$ bosons are vector bosons.

- $H \to W^+W^-$ decays produce pairs of W bosons in a singlet spin state
- In the narrow-width and non-relativistic approximations:

$$|\psi_s\rangle = \frac{1}{\sqrt{3}}\Big(|+\rangle\,|-\rangle - |0\rangle\,|0\rangle + |-\rangle\,|+\rangle \Big)$$

This is a Bell state

W bosons are their own polarimeters

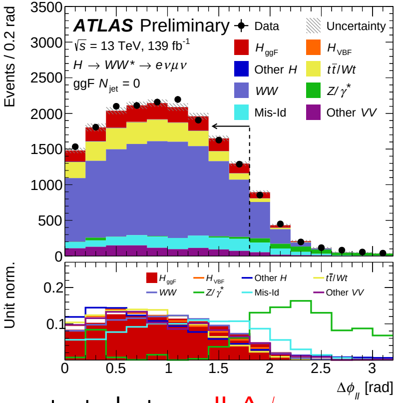
$V - A$ decays

SU(2) weak force is chiral

$$W^+ \to \ell^+_R + \nu_L$$
$$W^- \to \ell^-_L + \bar{\nu}_R$$

Decay of a $W^\pm$ boson is equivalent to a measurement of its spin along the axis of the emitted lepton

$\ell^+\ell^-$ azimuthal correlations in $H \to W^+W^-$



- Higgs signal concentrated at small $\Delta\phi_{\ell\ell}$
- Used e.g. in discovery searches

Quantum tests?

Entanglement

For some density matrix

$$\rho = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|$$

p_i is a classical probability

Q: Can we write:

$$\rho \stackrel{?}{=} \sum_i p_i \ \rho_A \otimes \rho_B \qquad p_i \geq 0, \sum p_i = 1$$

i.e. as a convex sum of product states?

- Yes $\Longrightarrow$ separable
- No $\Longrightarrow$ entangled

For general ρ (i.e. not pure states) this is a very different statement from just being correlated

Aside on pure states

Pure states are those for which ρ can be written:

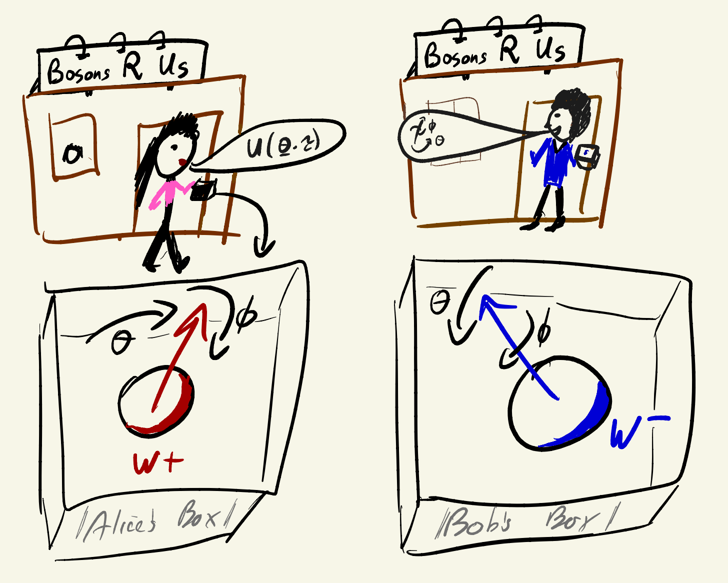
$$\rho = |\psi\rangle \langle\psi|$$

These idealised states have very particular properties. Consider, for example:

$$|\psi\rangle = \alpha\, |\uparrow_A\rangle \otimes |\uparrow_B\rangle + \beta\, |\downarrow_A\rangle \otimes |\downarrow_B\rangle$$

This is both entangled and correlated for $(\alpha,\ \beta) \neq 0$

But for a general ρ correlated $\neq$ entangled



You can't entangle particles with local operations and classical communication (LOCC)

Alice and Bob can make states like

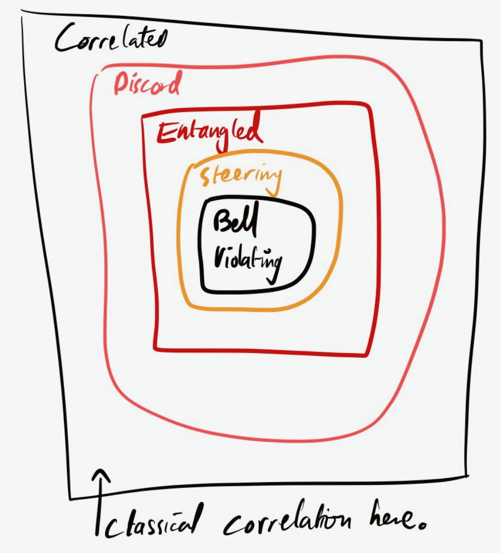
$$\rho_{\text{corr}} = \frac{1}{2}\Big(\rho_A(\uparrow) \otimes \rho_B(\uparrow) + \rho_A(\downarrow) \otimes \rho_B(\downarrow)\Big)$$

where

$$\rho_A(\uparrow) \equiv |\uparrow_A\rangle \langle\uparrow_A|$$

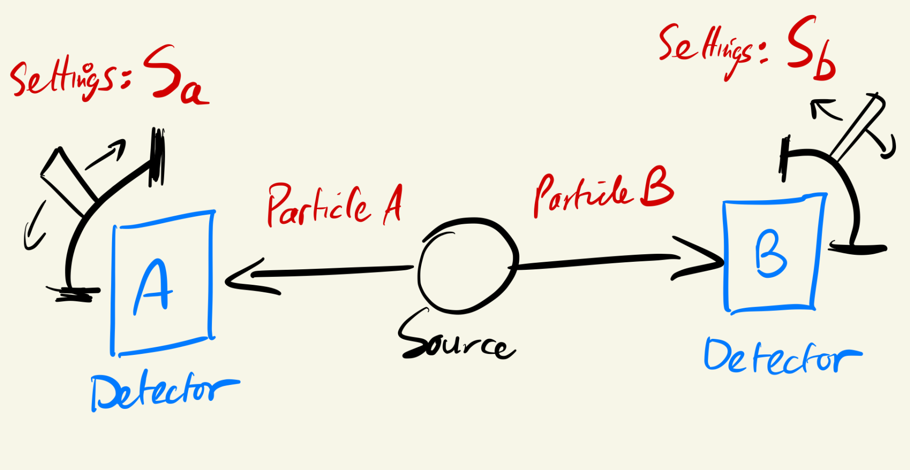
etc.

This is classically correlated, but not entangled – it can be written as a sum of products (as it is above)



For steering, discord see e.g. Y. Afik, J. de Nova 2209.03969

Bell inequality tests



The local realism formalism

Assume that there is a well-defined correlation function for the pair of measurement outcomes:

$$P(S_A, S_B) \equiv \int \mathrm{d}\vec{\lambda} \;\; a(S_A, \vec{\lambda}) \;\; b(S_B, \vec{\lambda}) \;\; P(\vec{\lambda})$$

May depend on 'hidden' variables $\vec{\lambda}$ which have a PDF $P(\vec{\lambda})$

Assumptions

- $a(S_A, \vec{\lambda})$ does **not** depend on S_B
- $b(S_B, \vec{\lambda})$ does **not** depend on S_A
- $P(\vec{\lambda})$ does **not** depend on S_A nor on S_B

Demand that marginal probabilities for measurements of A and B are non-negative

The CHSH Bell inequality

Clauser, Horne, Shimony & Holt (1969)

- The two experiments, A and B, each have two possible outcomes:
  { $+1$ or -1 }
  $E(a, b)$ is the expectation value of the product
- Each experiment has two possible settings :
  { primed or unprimed }
- Calculate the following function of the correlated expectations:

$$\mathcal{I}_2 = E(a, b) - E(a, b') + E(a', b) + E(a', b')$$

The CHSH Bell inequality

$$\mathcal{I}_2 = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

$$\text{Local realism} \implies |\mathcal{I}_2| \leq 2$$

Parameterise ρ – bipartite system of qubits

in terms of the Pauli matrices σ_i

Single qubit

$$\rho = \frac{1}{2} I_2 + \sum_{i=1}^{3} a_i \sigma_i,$$

a_i : 3 real parameters $(2^2 - 1)$

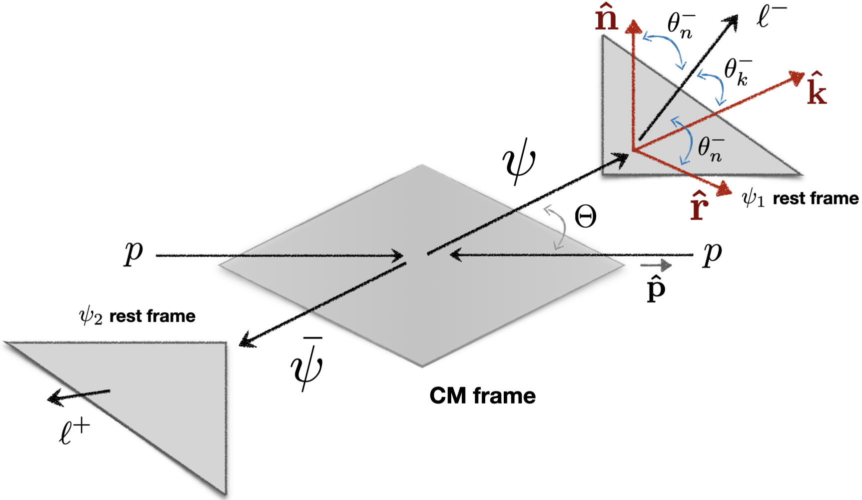
Two qubits

$$\rho = \frac{1}{4} I_2 \otimes I_2 + \sum_{i=1}^{3} a_i\, \sigma_i \otimes \frac{1}{2} I_2 + \sum_{j=1}^{3} b_j\, \frac{1}{2} I_2 \otimes \sigma_j + \sum_{i,j=1}^{3} c_{ij}\, \sigma_i \otimes \sigma_j,$$

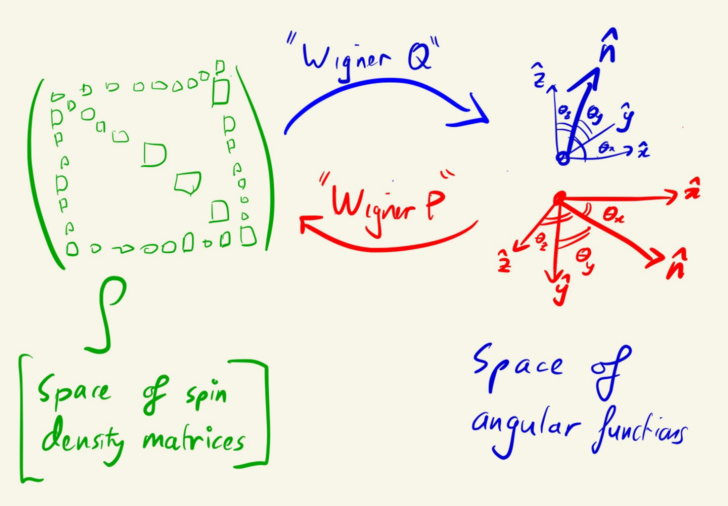
3+3+9 = 15 real parameters $(4^2 - 1)$

Measure the parameters $(a_i\ b_j, c_{ij})$ and test properties of bipartite ρ

Geometry



$\hat{\mathbf{n}}$
θ_n^-
ℓ^-
θ_k^-
$\hat{\mathbf{k}}$
ψ
θ_n^-
$\hat{\mathbf{r}}$
ψ_1 rest frame
Θ
p
p
$\hat{\mathbf{p}}$
ψ_2 rest frame
$\bar{\psi}$
CM frame
ℓ^+



Also true for e.g. $W^{\pm}$, Z^0, t , τ

Transforming between the spaces

The Wigner-Weyl formalism for spin

Operator $\rightarrow$ function

$$\Phi_A^Q(\hat{\mathbf{n}}) = \langle \hat{\mathbf{n}} | A | \hat{\mathbf{n}} \rangle$$

Wigner Q symbols

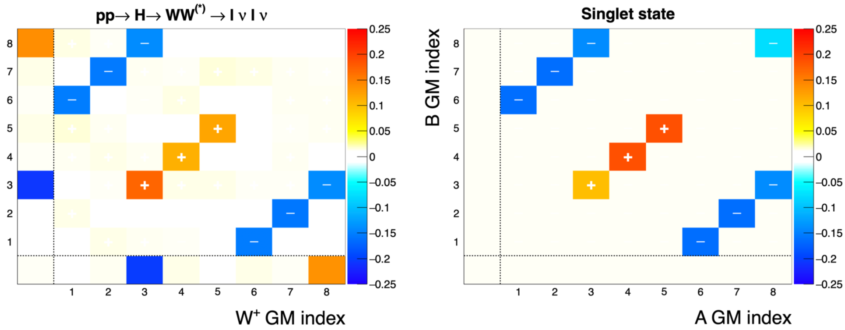
Function $\rightarrow$ operator

$$A = \frac{2j+1}{4\pi} \int \mathrm{d}\Omega_{\hat{\mathbf{n}}} \; |\hat{\mathbf{n}}\rangle \, \Phi_A^P(\hat{\mathbf{n}}) \, \langle \hat{\mathbf{n}}| \, ,$$

Wigner P symbols

Quantum State Tomography example

$H \to WW^*$ decays – qutrit pair



Density matrix parameters from simulated Higgs boson decays to vector bosons (Madgraph, no background)

LHC measurements

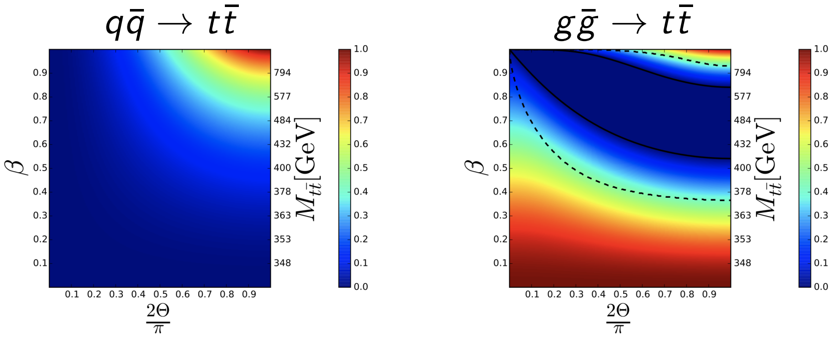


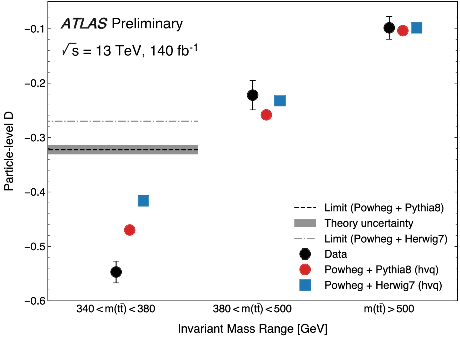
Figure 3: Concurrence of the spin density matrix $\rho^I(\beta, \hat{k})$ resulting from an initial state $I = q\bar{q}, gg$ as a function of the top velocity β and the production angle Θ in the $t\bar{t}$ c.m. frame. All plots are symmetric under the transformation $\Theta \to \pi - \Theta$. Left: $q\bar{q} \to t\bar{t}$. Right: $gg \to t\bar{t}$. Solid black lines represent the critical boundaries between separability and entanglement $\beta^{\mathrm{PH}}_{c1,c2}(\Theta)$, while dashed black lines represent the critical boundaries for the violation of the CHSH inequality, $\beta^{\mathrm{CH}}_{c1,c2}(\Theta)$.

Expect $t\bar{t}$ are entangled near threshold and at high p_T

Afik and de Nova: 2203.05582

Highest-energy detection of quantum entanglement

- $t\bar{t}$ spin-qubit pair
- Decay before hadronisation
- Leptons measure top spin
- $D = -\ tr[C]/3$
- $\exists$ no separable states with $D < -\frac{1}{3}$

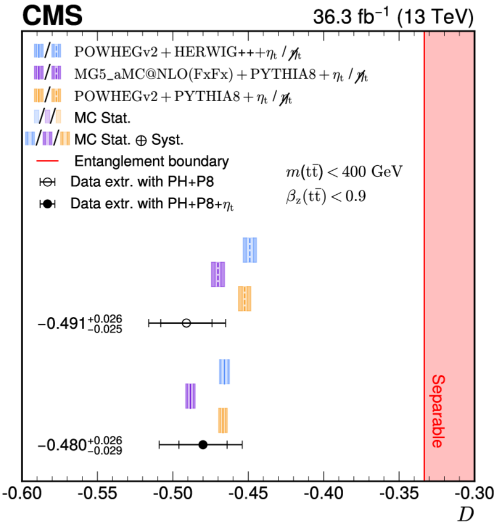


ATLAS result

$$D_{\mathrm{obs}} = -0.547 \pm 0.002\ [\mathrm{stat.}] \pm 0.021\ [\mathrm{syst.}] \qquad (> 5\sigma)$$

ATLAS: Briefing / ATLAS-CONF-2023-069 / 2311.07288

CMS near-threshold result

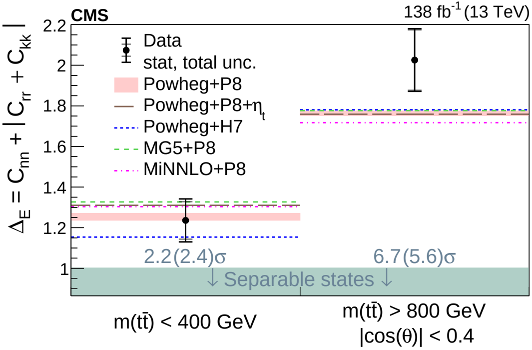


- Includes colour singlet toponium model
- $D = -0.478^{+0.025}_{-0.027}$
- 5.1 obs (4.7 exp) σ

CMS: Briefing / CMS-PAS-TOP-23-001 / 2406.03976

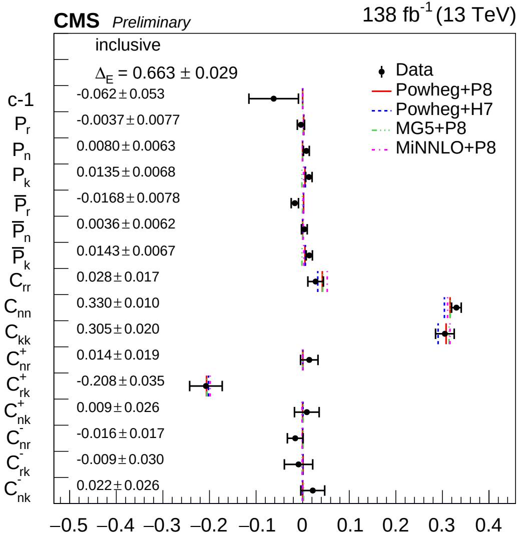
High-m_{tt} CMS result

- Semi-leptonic channel
- High invariant mass region
- t and $\bar{t}$ in spin triplet
- 90% of $t\bar{t}$ decays are space-like separated



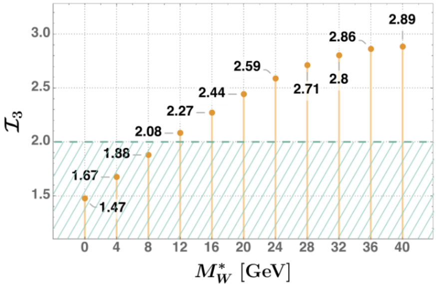
CMS: Briefing PAS 2409.11067

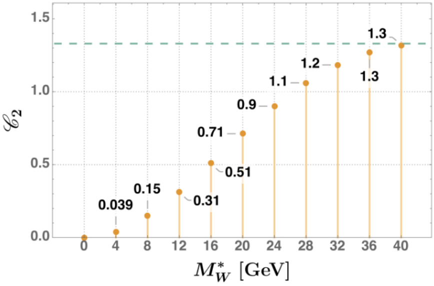
Full bipartite density matrix



Other possible measurements

H$\longrightarrow$ WW^*



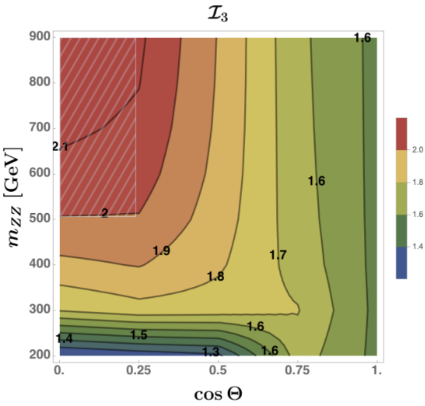


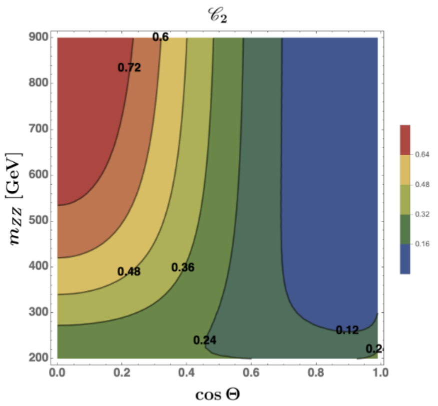
Optimised Bell Operator
> 2?

Bound on the concurrence
> 0?

Fabbrichesi et al. 2302.00683

pp$\longrightarrow ZZ$



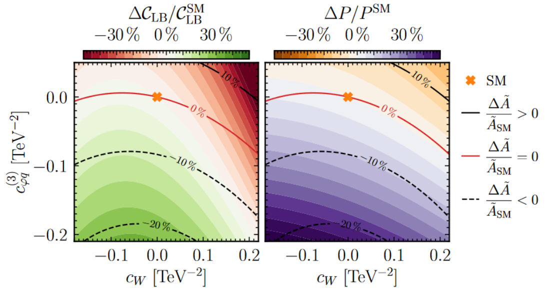


Optimised Bell Operator
$> 2?$

Bound on the concurrence
$> 0?$

Fabbrichesi et al. 2302.00683

Searching Beyond the Standard Model?



concurrence purity

- Production of $W\pm/Z$ pairs at pp, e^+e^-
- Quantum spin observables complementary probes of Wilson coefficients/EFT
- Offer increased sensitivity to certain operators

Aoude, Madge, Maltoni, Mantani *Probing new physics through entanglement in diboson production* 2307.09675

Many systems of interest

Even when just testing spin

Qubit systems

$\eta_c \to \Lambda + \bar{\Lambda}$

$pp \to t\bar{t}$

$e^+e^- \to \gamma^*/Z \to \tau^+\tau^-$

$h \to \tau^+\tau^-$

$h \to \gamma\gamma$

Qutrit systems

$B^0 \to J/\psi\, K^{*0}$

$B_s \to \phi\phi$

$pp \to WW \,/\, ZZ$

$h \to WW^* \,/\, ZZ^*$

Prospects at flavour factories, LHC, future e^+e^-, ...

A broad new programme for collider physics

Testing the foundations of quantum theory (and beyond?)

- 12 orders of magnitude higher energy that existing tests (shorter time scale, shorter length scale. . .)
- In 'self-measuring' quantum system
- Deep in the realm of quantum field theory (virtual particles)
- in qubit and qutrit systems
- in bipartite and tripartite systems

It's also a good way to find new fields

Many clever techniques and ideas being developed
Many measurements within reach (soon)

Review: AJB, M.Fabbrichesi, R.Floreanini, E.Gabrielli, L.Marzola: 2402.07972

EXTRAS

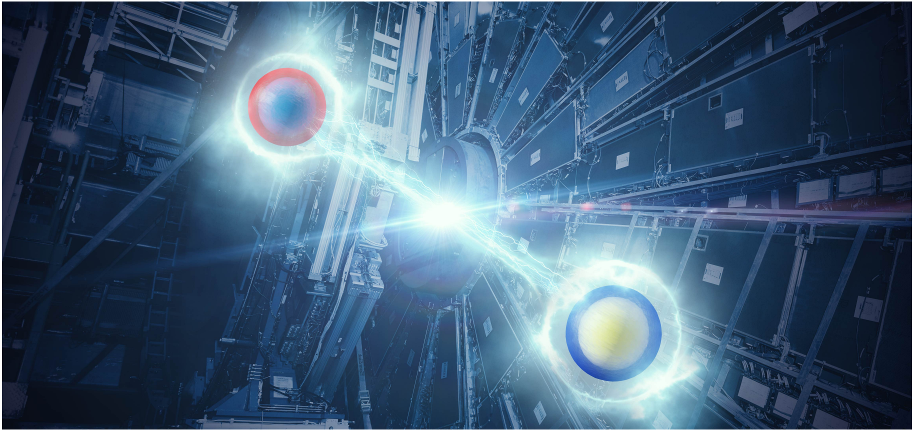


Image from ATLAS physics briefing